Exercise 75

A pen in the shape of an isosceles right triangle with legs of length x ft and hypotenuse of length h ft is to be built. If fencing costs 5/ft for the legs and 10/ft for the hypotenuse, write the total cost C of construction as a function of h.

Solution

Multiply the given lengths by the respective costs of construction and then add them together to get the total.

$$C = 5x + 10h$$

Because this is a right triangle, the sides are related by the Pythagorean theorem.

$$x^{2} + x^{2} = h^{2}$$
$$2x^{2} = h^{2}$$
$$x^{2} = \frac{h^{2}}{2}$$
$$x = \frac{h}{\sqrt{2}}$$
$$x = \frac{h\sqrt{2}}{2}$$

Therefore, the cost of construction as a function of h is

$$C(h) = 5x + 10h$$
$$= 5\left(\frac{h\sqrt{2}}{2}\right) + 10h$$
$$= \left(\frac{5\sqrt{2}}{2} + 10\right)h$$
$$= \frac{5}{2}(\sqrt{2} + 4)h$$
$$\approx 13.5h.$$